Energy Pricing and Dispatch under Demand Response and Market Price Uncertainty

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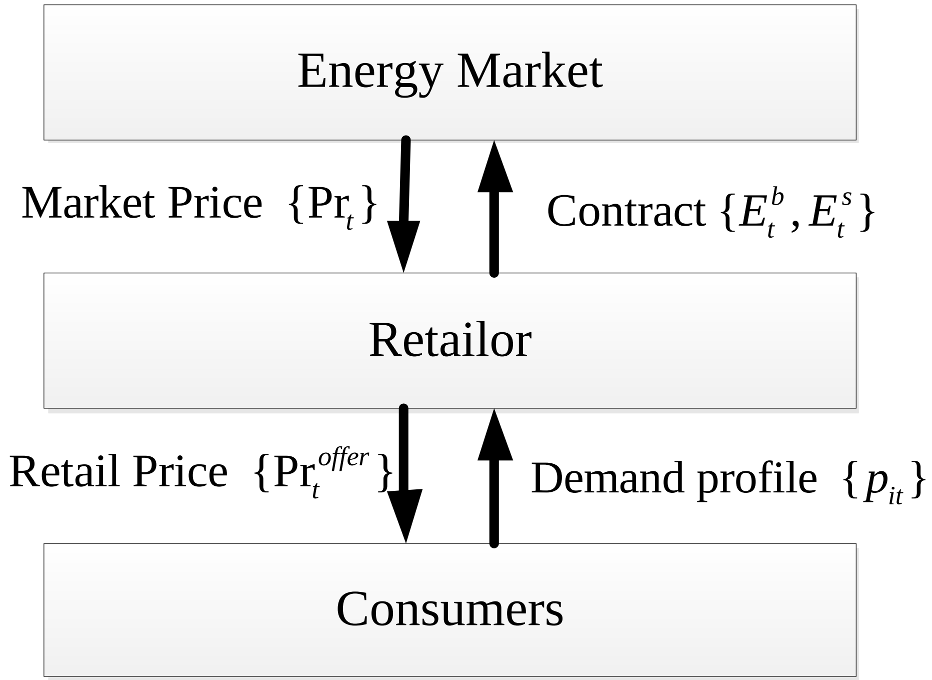
1. Structure of the retail market



- The retailor purchases energy at a wholesale market as a price-taker, and sells the energy to consumers as a price-maker.
- Retailer owns a storage unit.
- Energy price at the wholesale market is uncertain.
- Consumers adjust their demands according to the retail price.

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2. Mathematical formulation

A two-stage two-level decision framework

Stage 1: Day-ahead — Setting retail price

Stage 2: Real-time — Energy dispatch

Upper level — Retailer

$$\max_{\boldsymbol{f},\boldsymbol{y}} \sum_i \boldsymbol{f}^T \boldsymbol{y}^i(\boldsymbol{f})$$
$$s.t. \quad \boldsymbol{f} \in P$$

$$+ \quad \min_{\boldsymbol{c} \in W} \max_{\boldsymbol{x}} \boldsymbol{c}^T \boldsymbol{x}$$
$$s.t. \quad \boldsymbol{Ax} + \boldsymbol{By} \le \boldsymbol{b}$$

Retailer → Consumers: $\boldsymbol{p}$

Consumers → Retailer: $\boldsymbol{y}^i$

Consumers → Stage 2 (real-time energy dispatch): $\boldsymbol{y}^i$

Lower level — Consumers

$$\boldsymbol{y}^i = \arg\min_{\hat{y}} \boldsymbol{f}^T \hat{\boldsymbol{y}}^i$$
$$s.t. \; \hat{\boldsymbol{y}}^i \in C_i^P$$

2. Mathematical formulation

$$\max_{\Pr^{offer}\in P,\{p_{it}\}\in O(\Pr^{offer})} \sum_{t=1}^{N_T}\sum_{i=1}^{N_C} \Pr_t^{offer}\ p_{it} +$$

$$\min_{\Pr\in W}\ \max_{\{E^b,E^s,r^c,r^d,S\}\in X} \sum_{t=1}^{N_T}(\Pr_t-\varepsilon)E_t^s-(\Pr_t+\varepsilon)E_t^b$$

$$P=\left\{\Pr^{offer}\left|\begin{array}{c}\Pr_t^{\min}\le\Pr_t^{offer}\le\Pr_t^{\max},\quad\forall t\\ \sum_{t=1}^{N_T}\Pr_t^{offer}\big/N_T\le\Pr_{av}^f\end{array}\right.\right\}$$

$$W=\left\{\Pr\left|\begin{array}{c}\Pr_t^l\le\Pr_t\le\Pr_t^u,\forall t\\ \sum_t(\Pr_t-\Pr_t^f)\big/\Pr_t^v=0\\ \sum_t\left|\Pr_t-\Pr_t^f\right|\big/\Pr_t^v\le\Gamma\end{array}\right.\right\}$$

2. Mathematical formulation

$$O(\mathrm{Pr}^{offer}) = \{\boldsymbol{p} = \{\boldsymbol{p}^{i} = \{p_{it}\}, \forall t\}, \forall i \mid$$

$$\boldsymbol{p}^{i} = \arg\min_{\hat{p}_{it} \in C_i^P} \sum_{t=1}^{N_T} \mathrm{Pr}_t^{offer} \hat{p}_{it}$$

$$C_i^P = \{\hat{\boldsymbol{p}}^{i} \mid \hat{p}_{it} \leq P_i^m,\ \forall t \in T_a \ :\ \rho_{it}$$

$$\hat{p}_{it} \geq 0, \forall t \in T_a,\ \hat{p}_{it} = 0, \forall t \notin T_a \ :\ \theta_{it}$$

$$\sum_t \hat{p}_{it} = Q_i^d, \quad \mu_i \ \}$$

$$X = \{(\boldsymbol{E}^b, \boldsymbol{E}^s, \boldsymbol{r}^c, \boldsymbol{r}^d, \boldsymbol{S}) \mid$$

$$0 \leq r_t^c \leq ST_c^m, 0 \leq r_t^d \leq ST_d^m, \forall t,\ 0 \leq S_t \leq ST_c^e,\ \forall t \in \{2, \ldots, \ N_T - 1\}$$

$$S_t = S_{t-1} + \eta^c r_t^c - r_t^d / \eta^d,\ \forall t \in \{2, \ldots, N_T\},\ S_t = ST_0^e,\ \ t \in \{1, N_T\}$$

$$\sum_{i=1}^{N_C} p_{it} + r_t^c - r_t^d = E_t^b - E_t^s,\ \forall t,\ \ E_t^b \geq 0,\ E_t^s \geq 0,\ \ \forall t \ \}$$

3. Solution method

A polyhedral representation of W

$$W = \left\{ \Pr \left| \begin{array}{c} \Pr_t^l \le \Pr_t \le \Pr_t^u, \forall t \\ \sum_t (\Pr_t - \Pr_t^f) / \Pr_t^v = 0 \\ \sum_t \left| \Pr_t - \Pr_t^f \right| / \Pr_t^v \le \Gamma \end{array} \right. \right\}$$

$$= \left\{ \Pr \left| \begin{array}{c} \Pr_t^l \le \Pr_t \le \Pr_t^u, \forall t \ : \ \eta_t^l, \eta_t^u \\ \sum_t (\Pr_t - \Pr_t^f) / \Pr_t^v = 0 \ : \ \xi \\ \exists \boldsymbol{u}, \ \sum_{t=1}^{N_T} u_t \le \Gamma \ : \ \varsigma \\ -u_t \le (\Pr_t - \Pr_t^f) / \Pr_t^v \le u_t, \forall t \ : \ \lambda_t^l, \lambda_t^u \end{array} \right. \right\}$$

3. Solution method

Second stage problem transformation

$$\min_{\boldsymbol{c}} \max_{\boldsymbol{x}} \boldsymbol{c}^T \boldsymbol{x}, \quad c = [-\mathbf{Pr} - \mathbf{1}\varepsilon, \mathbf{Pr} + \mathbf{1}\varepsilon, \mathbf{0}]$$

$$s.t.\ \boldsymbol{D}\mathbf{Pr} + \boldsymbol{F}\boldsymbol{u} \geq \boldsymbol{d}, \quad \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b} - \boldsymbol{B}\boldsymbol{y}$$

For a fixed bidding strategy, the worst-case market price solves an LP

$$\begin{aligned} &\min_{\boldsymbol{c}} \boldsymbol{c}^T \boldsymbol{x} = \\ &\min_{\mathrm{Pr},u} (\boldsymbol{x}_2 - \boldsymbol{x}_1)\mathbf{Pr} \\ &s.t.\ \boldsymbol{D}\mathbf{Pr} + \boldsymbol{F}\boldsymbol{u} \geq \boldsymbol{d} \end{aligned} \quad \overset{\text{Dual}}{=====>} \quad \begin{aligned} &\max \boldsymbol{d}^T \boldsymbol{\lambda} \\ &s.t.\ \boldsymbol{F}^T \boldsymbol{\lambda} = \mathbf{0}, \boldsymbol{\lambda} \geq \mathbf{0} \\ &\boldsymbol{D}^T \boldsymbol{\lambda} = \boldsymbol{x}_2 - \boldsymbol{x}_1 \end{aligned}$$

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3. Solution method

Second stage problem transformation

The linear min-max problem is equivalent to an LP

$$\left\{\begin{array}{c}\min_{\boldsymbol{c}}\max_{\boldsymbol{x}} \boldsymbol{c}^T\boldsymbol{x} \\ s.t.\ \boldsymbol{D}\mathbf{Pr}+\boldsymbol{F}\boldsymbol{u}\geq\boldsymbol{d} \\ \boldsymbol{A}\boldsymbol{x}\leq\boldsymbol{b}-\boldsymbol{B}\boldsymbol{y} \\ c=[-\mathbf{Pr}-\mathbf{1}\varepsilon,\mathbf{Pr}+\mathbf{1}\varepsilon,\mathbf{0}]\end{array}\right\}=\left\{\begin{array}{c}\max_{x,\lambda}\boldsymbol{d}^T\boldsymbol{\lambda} \\ s.t.\ \boldsymbol{F}^T\boldsymbol{\lambda}=\mathbf{0},\boldsymbol{\lambda}\geq\mathbf{0} \\ \boldsymbol{D}^T\boldsymbol{\lambda}=\boldsymbol{x}_2-\boldsymbol{x}_1 \\ \boldsymbol{A}\boldsymbol{x}\leq\boldsymbol{b}-\boldsymbol{B}\boldsymbol{y}\end{array}\right\}$$

- This conclusion holds for polyhedral uncertainty in objective function.

3. Solution method

Lower-level problem transformation

$$\min \boldsymbol{p}^{iT} \mathbf{Pr}^{offer}$$
$$s.t. \boldsymbol{H}^{i} \boldsymbol{p}^{i} \leq \boldsymbol{h}^{i}$$

primal-dual optimality conditions

$$\mathbf{Pr}^{offer} \boldsymbol{p}^{i} = \boldsymbol{h}^{T} \boldsymbol{\mu}$$
$$\boldsymbol{H}^{i} \boldsymbol{p}^{i} \leq \boldsymbol{h}^{i}$$
$$\boldsymbol{\mu}^{i} \leq \mathbf{0},\ \boldsymbol{H}^{iT} \boldsymbol{\mu}^{i} = \mathbf{0}$$

The optimization problem can be replaced by explicit constraints

3. Solution method

Linearizing bilinear terms

Apply binary expansion on the retail price

$$\Pr_t^{offer} = \Pr_t^{\min} + \Delta_t^s \sum_{l=1}^{K} 2^{l-1} z_t^l,\ \forall t$$

Introduce new variable $v_{it}^l = p_{it} z_t^l,\ \forall i,t$

The bilinear terms can be linearized as

$$\Pr_t^{offer}\ p_{it} = \Pr_t^{\min}\ p_{it} + \Delta_t^s \sum_{l=1}^{K} 2^{l-1} v_{it}^l,\ \forall i,t$$

$$0 \le p_{it} - v_{it}^l \le P_i^m (1 - z_t^l),\ \forall i,t,l$$

$$0 \le v_{it}^l \le P_i^m z_t^l,\ \forall i,t,l,\quad z_t^l \in \{0,1\},\ \forall t,l$$

The number of binary variables are independent of the number of consumers

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3. Solution method

The Equivalent MILP

$$\max \sum_{i=1}^{N_C} (\boldsymbol{p}^{iT} \mathbf{Pr}^{\min} + \mathbf{1}^T \boldsymbol{V}^i \boldsymbol{\Delta}) + \boldsymbol{d}^T \boldsymbol{\lambda}$$

$$s.t.\ \{\mathbf{Pr}^{offer} = \mathbf{Pr}^{\min} + \mathbf{Z}\boldsymbol{\Delta}\} \in P$$

$$\boldsymbol{F}^T \boldsymbol{\lambda} = \mathbf{0}, \boldsymbol{\lambda} \geq \mathbf{0}, \boldsymbol{D}^T \boldsymbol{\lambda} = \boldsymbol{x}_2 - \boldsymbol{x}_1$$

$$\boldsymbol{Ax} \leq \boldsymbol{b} - \boldsymbol{By},\ \boldsymbol{y} = \{\boldsymbol{p}^i\}, \forall i$$

$$\mathbf{Pr}^{\min} \boldsymbol{p}^i + \mathbf{1}^T \boldsymbol{V}^i \boldsymbol{\Delta} = \boldsymbol{h}^{iT} \boldsymbol{\mu}^i$$

$$\boldsymbol{H}^i \boldsymbol{p}^i \leq \boldsymbol{h}^i, \boldsymbol{\mu}^i \leq \mathbf{0},\ \boldsymbol{H}^{iT} \boldsymbol{\mu}^i = \mathbf{0}$$

$$\mathbf{0} \leq \boldsymbol{p}^i \mathbf{1}^T - \boldsymbol{V}^i \leq P_i^m (\mathbf{1} - \boldsymbol{Z}),\ \forall i$$

$$\mathbf{0} \leq \boldsymbol{V}^i \leq P_i^m \boldsymbol{Z},\ \forall i,\ \boldsymbol{Z} \in \{0,1\}^{T \times L}$$

3. Solution method

Pareto efficiency test

- The risk-aversive dispatch strategy only accounts for the worst-case market price realization
- There may be another strategy that can improve the retailer's profit for at least one scenario of market price without deteriorating it in all other scenarios.

$$PE = \max_{\gamma^1,\gamma^2,\gamma^3,\gamma^4,\gamma^5} \; (\mathrm{Pr}^f - \mathbf{1}\varepsilon)^T \boldsymbol{\gamma}^1 - (\mathrm{Pr}^f + \mathbf{1}\varepsilon)^T \boldsymbol{\gamma}^2$$

$$s.t. \quad (\boldsymbol{\gamma}^1, \boldsymbol{\gamma}^2) \in W^*$$

$$\{\boldsymbol{E}_*^b + \boldsymbol{\gamma}^1, \boldsymbol{E}_*^s + \boldsymbol{\gamma}^2, \boldsymbol{r}_*^c + \boldsymbol{\gamma}^3, \boldsymbol{r}_*^d + \boldsymbol{\gamma}^4, \boldsymbol{S}_* + \boldsymbol{\gamma}^5\} \in X$$

3. Solution method

Pareto efficiency test

*W** is the dual cone of the polytope *W*

$$W^* = \{(\boldsymbol{\gamma}^1, \boldsymbol{\gamma}^2) \mid (\mathrm{Pr} - \mathbf{1}\varepsilon)^T \boldsymbol{\gamma}^1 - (\mathrm{Pr} + \mathbf{1}\varepsilon)^T \boldsymbol{\gamma}^2 \geq 0,\ \forall \mathrm{Pr} \in W\}$$

$$= \{(\boldsymbol{\gamma}^1, \boldsymbol{\gamma}^2) \mid \exists \{\eta_t^l, \eta_t^u, \lambda_t^l, \lambda_t^u, \xi, \varsigma\}, \text{such that}$$

$$\eta_t^l \geq 0,\ \eta_t^u \leq 0,\ \lambda_t^l \geq 0,\ \lambda_t^u \leq 0,\ \forall t,\ \varsigma \leq 0$$

$$\gamma_t^1 - \gamma_t^2 - \eta_t^l - \eta_t^u - \lambda_t^l / \mathrm{Pr}_t^v - \lambda_t^u / \mathrm{Pr}_t^v - \xi / \mathrm{Pr}_t^v = 0, \forall t$$

$$-\lambda_t^l + \lambda_t^u - \varsigma = 0,\ \forall t$$

$$\sum_{t=1}^{N_T} (\mathrm{Pr}_t^l \eta_t^l + \mathrm{Pr}_t^u \eta_t^u + (\lambda_t^l + \lambda_t^u) \mathrm{Pr}_t^f / \mathrm{Pr}_t^v)$$

$$+\xi \sum_t \mathrm{Pr}_t^f / \mathrm{Pr}_t^v + \varsigma\Gamma \geq \varepsilon \sum_{t=1}^{N_T} (\gamma_t^1 + \gamma_t^2)\ \}$$

3. Solution method

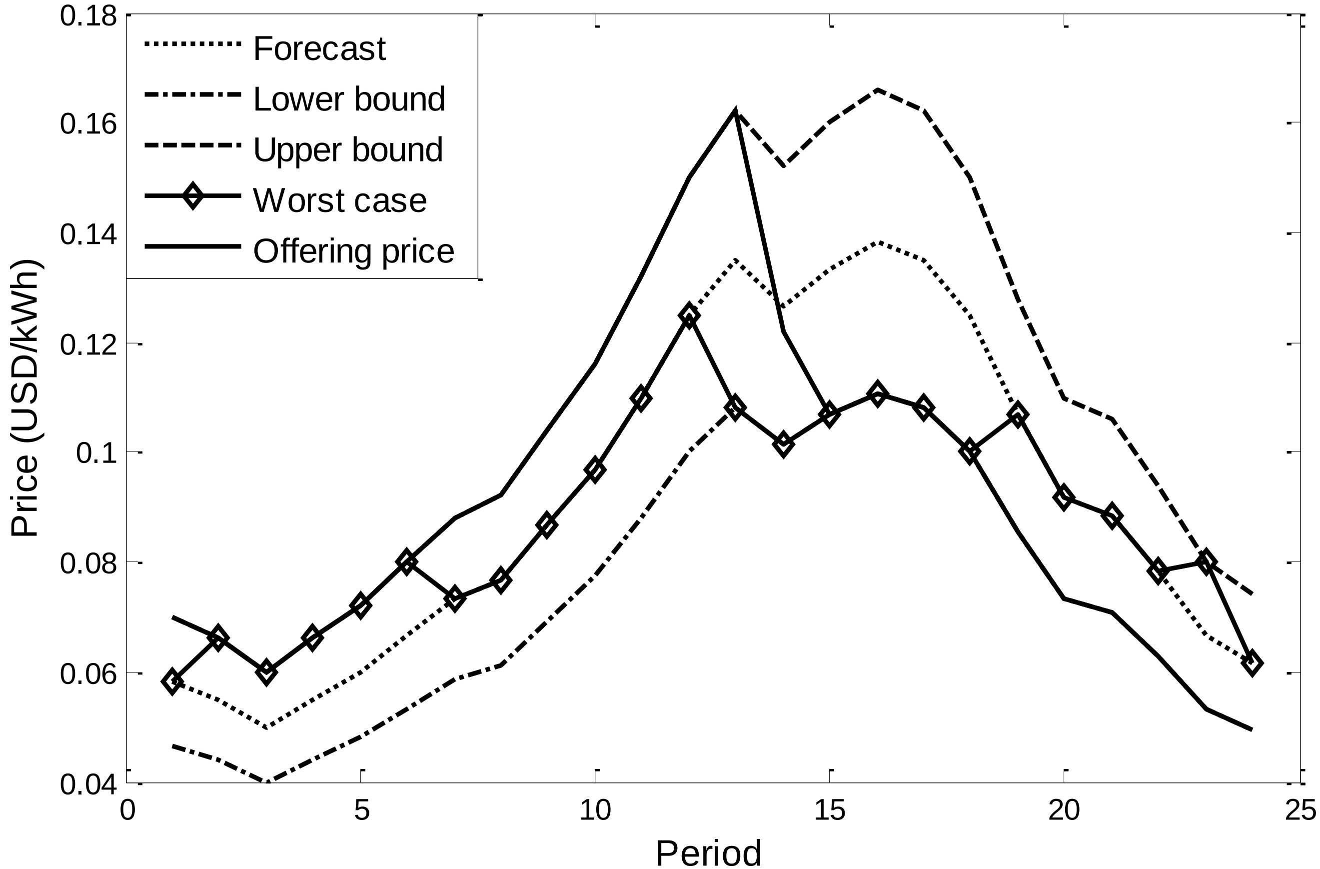
Pareto efficiency test

- Because 0 is a feasible solution, PE≥0.
- If PE = 0, the current dispatch strategy $x^* = \{E_*^b, E_*^s, r_*^c, r_*^d, S_*\}$ is already non-dominated.
- If PE > 0, $z^* = x^* + \gamma^*$ dominates x^* and z^* is non-dominated.

Iancu D A, Trichakis N. Pareto efficiency in robust optimization. Management Science, 2014, 60(1): 130-147.

4. Case studies

Retail and market price



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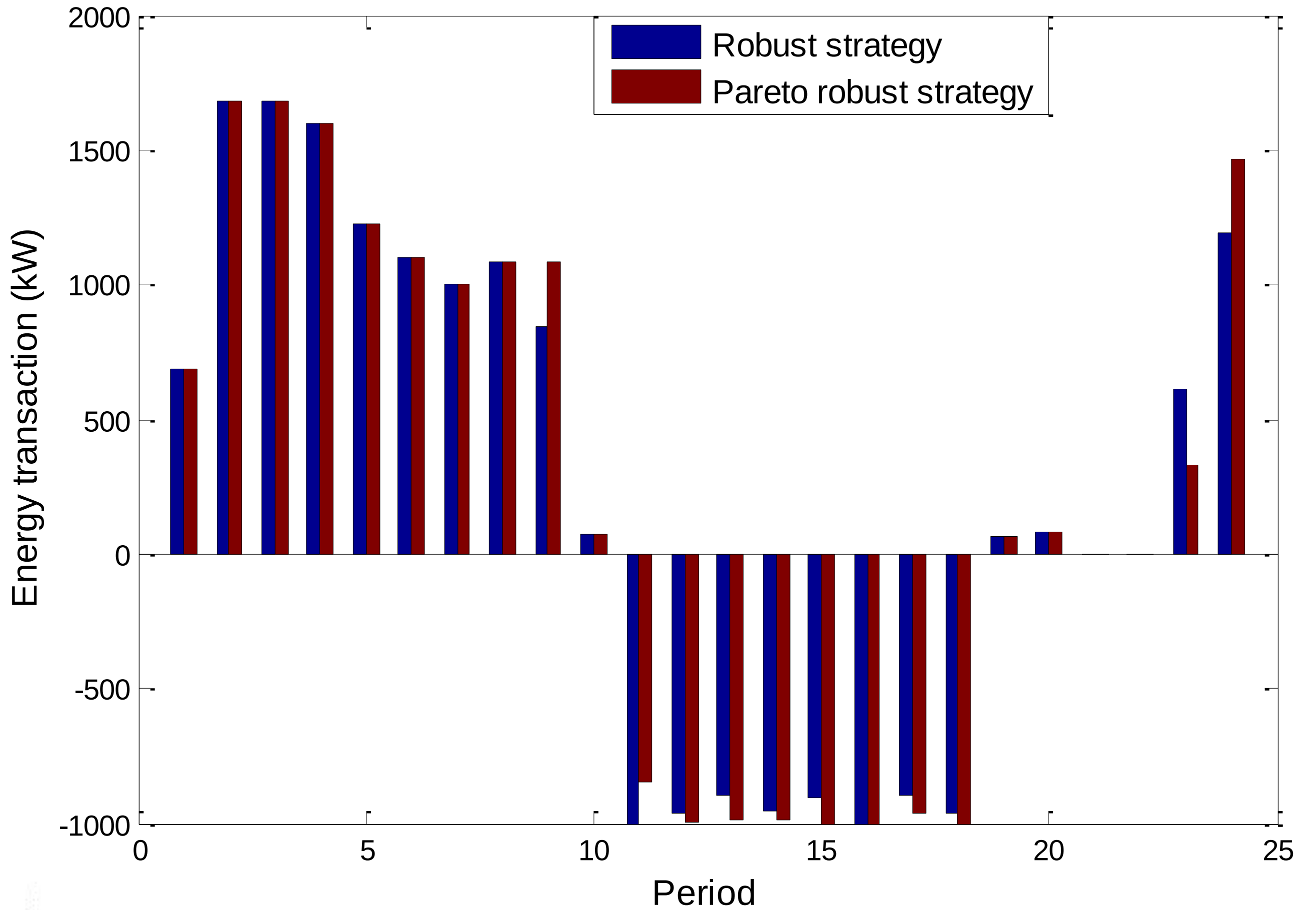
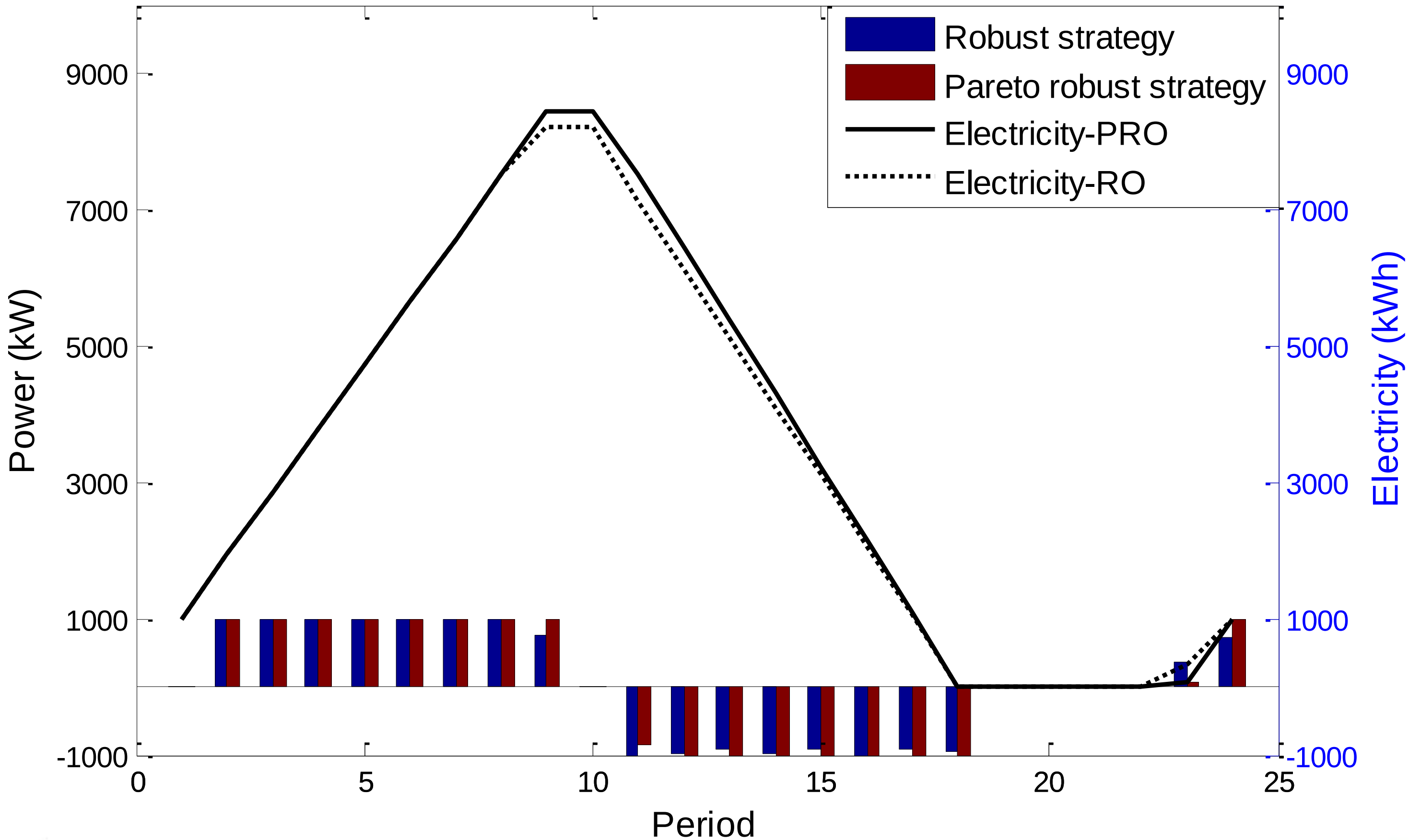
4. Case studies

Energy contract



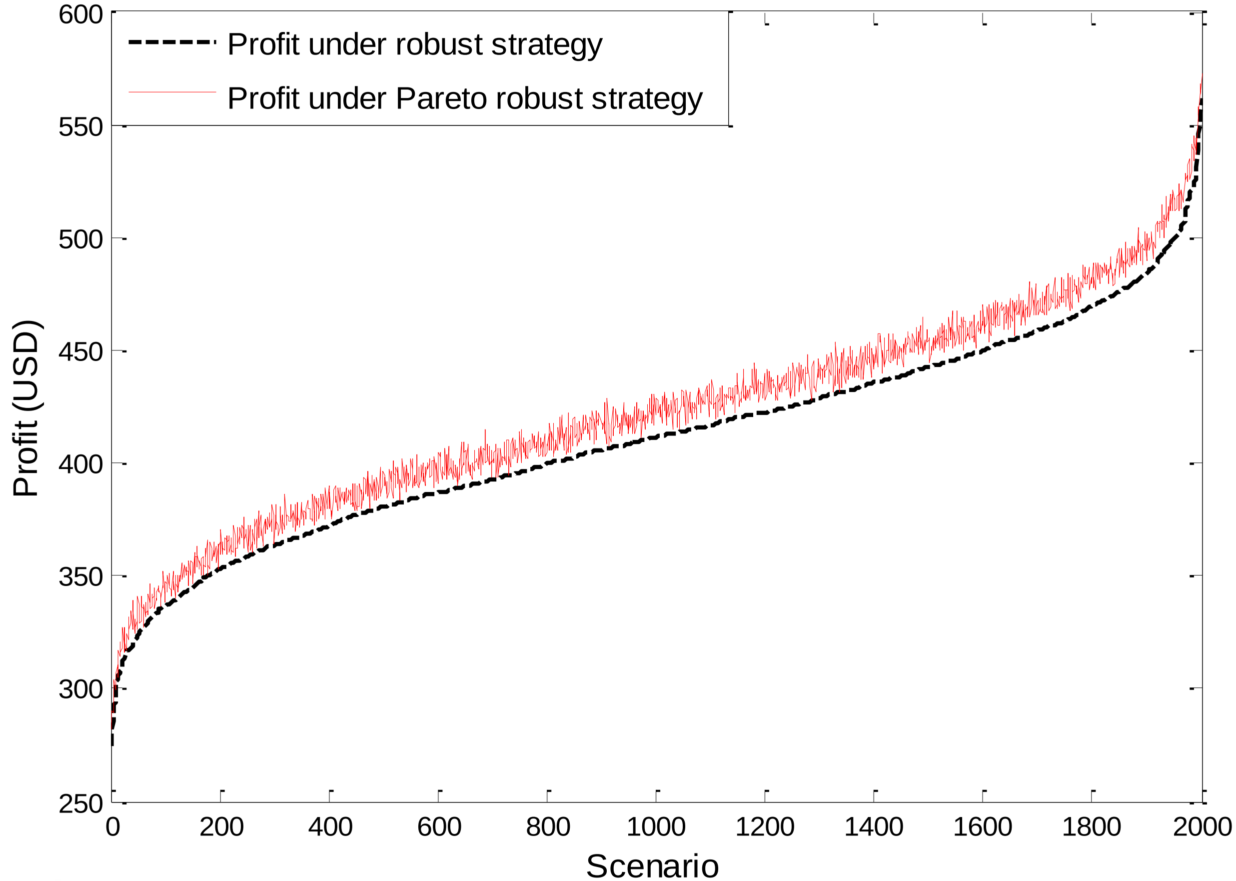
4. Case studies

Operation of storage unit



4. Case studies

Profit enhancement



Thanks!